The Journal of Law, Economics, and Organization, 2026, **00**, 1–31
https://doi.org/10.1093/jleo/ewaf020
Article

OXFORD

Adversarial disclosure with cross-examination

Claude Fluet[1,*], Thomas Lanzi[2]

[1]Department of Finance, Université Laval, Québec City, Canada
[2]Bureau d'Économie Théorique et Appliquée, Université de Lorraine, Nancy, France

[*]Corresponding author: E-mail: claude.fluet@fsa.ulaval.ca.

ABSTRACT

Two opposed parties seek to influence the court. They invest in gathering evidence and select what to disclose. The court then adjudicates. We compare this setting with one allowing cross-examination. A cross-examiner tests the opponent to persuade the court that relevant information was withheld. Cross-examination without appropriate safeguards is shown to be detrimental to the quality of adjudication. It shifts the risk of adjudication error from erroneously finding for the cross-examined to erroneously finding for the cross-examining party. That party therefore has less incentives to acquire evidence because the opponent can now also be countered through cross-examination. A potential cross-examined also has less incentives to gather evidence because there is now a smaller chance of prevailing. Adjudication is therefore less informed. We show that redirect examination (or substitutes such as court-led examination) plays an essential role in correcting undesirable effects. Together, cross and redirect efficiently extract information from a testifying party (*JEL* C7, D8, K4).

1. INTRODUCTION

Cross-examination is a key feature of the common law trial. It refers to the post-testimony questioning of a witness brought forth by the opposing party. In a much-quoted phrase, cross-examination "is beyond any doubt the greatest legal engine ever invented for the discovery of truth." (Wigmore 1940: §1367) It has also been criticized: "Wigmore's celebrated panegyric … is nothing more than an article of faith." (Langbein 1985: 834) Most of the criticism focuses on the potential for "false positives." Judge Frankel remarked that a skillful cross-examiner "will employ ancient and modern tricks to make a truthful witness look like a liar." (Frankel 1980: 16) Indeed, a caveat shortly follows Wigmore's famous quotation: "A lawyer can do anything with cross-examination … He may … do more than he ought to do; he … may make the truth appear like falsehood" (Wigmore 1940: §1367).

The above abstracts from procedural safeguards, for example, cross-examination is tempered by judicial oversight, objections to how questions are framed can be raised by the counsel of the cross-examined or by the judge herself. In particular, the right of redirect examination (or re-examination) by the counsel of the cross-examined may correct or mitigate erroneous inferences. Beyond the common law trial, across legal systems and adjudicatory bodies, there is a variety of procedural rules governing examination, including nonpartisan examination by the bench, adversarial cross-examination but with the equivalent of redirect examination by the bench, restrictions on the scope of adversarial cross-examination and party-led re-examination, and up to no opportunity at all (Weigend 2010). More or less restricted modalities are frequent within decision-making entities such as industrial tribunals, disciplinary panels, and commercial arbitration boards. In the United States over recent years, a controversy developed about whether cross-examination should be allowed in Title IX hearings about sexual assault cases on college campuses. Opponents argued that cross-examination would deter reporting of sexual assaults, while proponents emphasized the right to due process for respondents. One proposal was to control cross-examination by requiring the accused party to submit cross-examining questions to a neutral examiner who would select the ones most likely to shed light on the case.[1]

It has been remarked that there is no theory—and little empirical evidence—on how cross-examination or re-examination work.[2] We develop a model where party-led examination plays a central role. Two opposed parties seek to influence a decision maker. At the end of the procedure, a binary decision will be made in favor of one party or the other. The parties invest in uncovering evidence to support their case. The potential evidence consists of pieces of hard information, and there is uncertainty about how many pieces there are. The intrinsic meaning of the evidence is common knowledge, and outright lies are not possible, but a party may be deceitful through withholding some of the evidence, that is, a party strategically selects what to disclose. Once a party has testified, cross-examination by the opponent enables the elicitation of information about the testifying party's potential dishonesty, allowing the decision maker to update her belief about the meaning of a submission. Cross-examiners adopt lines of questioning that will sometimes yield false positives, meaning that the testifying party "fails" cross-examination even though the party was honest. When the procedure allows it, redirect examination by the testifying party's own counsel then attempts to rehabilitate the testimony. The re-examiner's line of questioning will sometimes change false positives into true negatives, that is, an honest testimony is rehabilitated, but it will also sometimes change true positives into false negatives, that is, a dishonest testimony may also be rehabilitated.

We find that, if cross-examination is to be permitted, then so should re-examination. By comparison with a procedure without cross-examination, allowing cross-examination without the opportunity of re-examination tends to reduce the quality of decision-making. Adjudication errors increase given the adversarial nature of cross-examination and because of its effect on the gathering of evidence. Redirect examination, following cross-examination, mitigates the undesirable effects and is conducive to a more valuable examination outcome from the decision-maker's point of view.

To illustrate, consider a court procedure without cross-examination. The plaintiff bears the burden of proof and invests in gathering evidence to discharge the burden. For simplicity, suppose the defendant is passive in this respect, that is, the defendant has no access to

[1] In 2011, the Obama administration revised Title IX grievance procedures in order to encourage reporting. In 2020, the Trump administration issued its own guidance. The controversial new requirement concerned adversarial cross-examination. See Dowling (2021), and the references therein.

[2] Wigmore himself: "What is the theory of [cross-examination's] efficiency? ... Upon this we commonly reflect but little." (Dowling: §1368). See also Lempert: "[T]he likely effectiveness of cross-examination in getting at the truth is seldom examined—numerous court opinions and commentaries rely on Wigmore's conclusion ... rather than on empirical evidence." (Lempert 1998: 345).

the evidence. The adjudication error will then depend on the plaintiff's investigation effort and on the possible manipulation of the evidence, that is, withholding unfavorable pieces of evidence. Suppose the court is ready to find for the plaintiff, given the evidence submitted. The court understands that the plaintiff may have withheld relevant evidence, but it estimates that this risk is small enough. At this point, let us now introduce the opportunity of cross-examination. Because the defendant stands to lose the case, its counsel will cross-examine the plaintiff (or the plaintiff's experts or witnesses). With some probability, cross-examination will expose a deceitful plaintiff; it may also turn the tables against a non-deceitful plaintiff. We show that, given the strategic nature of cross-examination, the probability of adjudication error with the added cross-examination stage is the same as without cross-examination. The risk of error merely shifts (in part) from favoring the plaintiff to favoring the defendant. However, when the opportunity of cross-examination is anticipated, there is a chilling effect on the plaintiff's investment in gathering evidence because of the smaller chance of prevailing. As a result, the average adjudication error will be greater than without cross-examination.

Consider now the same setting but with cross-examination followed by the opportunity of redirect. Conditional on the evidence acquired by the plaintiff, the probability of adjudication error is now smaller than in the procedure without cross-examination, and it is also smaller than in the procedure with cross-examination alone. False positives then occur less often, and so do true positives, but the former effect dominates. We obtain that cross-examination combined with re-examination optimally extracts information from the plaintiff, in the sense of minimizing the probability of adjudication error conditional on the evidence acquired by the plaintiff. However, there is still a chilling effect on the plaintiff's incentives to gather evidence. The overall effect on the quality of adjudication therefore depends on the relative importance of the chilling effect compared to the information that can be extracted from the plaintiff.

Our article relates to several strands of literature. The starting point is the disclosure game literature where evidence is hard information that cannot be falsified or forged but can be concealed (Grossman 1981; Milgrom 1981; Milgrom and Roberts 1986). Information can then be manipulated when a party's information status is unobservable (Dye 1985; Shavell 1989). We integrate the possibility of cross-examination into this framework. Cross-examination is interpreted in the general sense of actions that seek to lessen the weight of another party's report, not by providing directly relevant countervailing evidence but by questioning the report's significance or interpretation.

Our basic set-up is similar to Shin's (1998) analysis of the adversarial procedure but with the following additional features. First, the parties' information is endogenous as in Kim (2014) or Kartik et al. (2017). Second, the information acquired by a party may consist of several pieces (Dewatripont and Tirole 1999; Demougin and Fluet 2008; Bull and Watson 2019). The two features together allow for equilibria where both parties invest in acquiring evidence. Moreover, when a party submits evidence, the decision maker may remain uncertain whether the party disclosed the whole truth. This allows for actions, which we refer to as cross-examination or re-examination that generates information about the eventual withholding of evidence. To capture the partisan nature of cross-examination, and similarly for redirect examination, we model examination as a "publicly observable experiment" in the spirit of the Bayesian persuasion literature (Kamenica and Gentzkow 2011; Bergemann and Morris 2019; Kamenica 2019). A cross-examiner subjects the cross-examined to a testing process in order to persuade the decision maker that the cross-examined concealed unfavorable information. A re-examiner seeks to repair the damages caused by cross-examination.

We assume that examiners are "constrained" information designers in the sense that a perfectly revealing examination is not feasible. If the withholding of evidence in a testimony could be detected without error through cross-examination, no party would have an incentive to conceal evidence. The informational constraints may be interpreted as stemming from the examiners' limited skills, the limited time allowed for examination, and the intrinsic complexities of the situation. A cross-examiner's test is therefore designed to maximize the probability that the cross-examined will fail the test, subject to Bayesian plausibility and the constraints defining the set of feasible tests. Re-examination is modeled similarly and is subject to the same informational constraints, except that the re-examiner has the opposite objective. When the procedure allows both cross and re-examination, there is persuasion and counterpersuasion as in models of Bayesian persuasion with multiple senders (Gentzkow and Kamenica 2017a, 2017b; Li and Norman 2021).

Section 2 presents the setting. Section 3 describes the equilibrium outcome under the different procedures, with and without the opportunity of cross-examination and re-examination. Section 4 compares the quality of adjudication under the different procedures. Section 5 discusses the relation to another strand of literature, mandatory disclosure and the right to silence, and concludes. Proofs are in Appendix A. Some extensions of our baseline settings are discussed in Appendix B.

2. MODEL

An arbitrator, denoted A, must settle an issue between two parties, plaintiff P and defendant D. The issue is the determination of $\omega \in \{\omega_0, \omega_1\}$ or true fact, where ω_1 favors the plaintiff. The arbitrator's decision is $d \in \{0,1\}$, her payoff is $u_A(d,\omega) = 1$ if $d = i$ when $\omega = \omega_i$, $i \in \{0,1\}$, and $u_A(d,\omega) = 0$ otherwise. While the arbitrator wants her decision to match the true fact, the parties only care about who benefits. The plaintiff's payoff from the decision is d, the defendant's payoff is $-d$. We say that P prevails when $d = 1$ and that D prevails when $d = 0$.

In our baseline setting, neither the plaintiff nor the defendant initially have private information about the true fact: the parties and the arbitrator share the same prior p that $\omega = \omega_1$. For instance, the dispute is about the determination of property rights or about causation, that is, whether some act by the defendant was harmful. In Appendix B, we briefly consider the important case where the issue is whether the defendant did some act, as in a tort case under the negligence rule. The defendant knows whether he was negligent, and this will affect his incentives to invest time and money to gather hard evidence.[3] We show that our results about the effects of cross and redirect examination nevertheless remain the same as in our baseline setting.

2.1. Investigation and reports

There is uncertainty about the body of evidence that could be brought forward, for example, related facts, documents, expert reports, or witness testimonies. With probability $\theta \in (0,1)$, the evidence contains two pieces of hard information, x and y with realizations in $\{x_0, x_1\}$ and $\{y_0, y_1\}$ respectively. With probability $1-\theta$, the evidence consists of a single piece x. We let M denote the state where the evidence is *Multiple* with the two pieces x and y; S is the state where the evidence consists of the *Single* piece x.

[3] In adversarial disclosure games with acquisition of evidence, it has been common to assume that the parties are similarly uninformed (for example, Dewatripont and Tirole 1999, Kim 2014, Kartik et al. 2017).

Searching for the evidence is costly and not always successful. Party $j \in \{P, D\}$ uncovers the evidence with probability e_j at a cost $C(e_j)$, an increasing and strictly convex function with $C(0) = C'(0) = 0$ and $C'(1) \geq 1$. The inequality ensures that, with the stakes normalized to unity, a party will never want to invest to the point of obtaining the evidence for sure. The parties' net payoffs are

$$u_P = d - C(e_P), \quad u_D = -d - C(e_D).$$

How the evidence relates to the issue is as follows. $P(x, y, \omega)$ is the joint probability of the true fact and of the evidence when the state of the evidence is M. We also use the notation P for marginal or conditional distributions. For instance, the prior is $P(\omega_1) = p$. Applying Bayes' rule, the posteriors are

$$P(\omega_i \mid x, y) = \frac{P(x, y, \omega_i)}{P(x, y, \omega_1) + P(x, y, \omega_0)} \quad i = 0, 1.$$

When the state of the evidence is S, the joint probability of x and ω is $P(x, \omega) = \sum_y P(x, y, \omega)$.

Assumption 1: For $i = 0, 1$, $P(y_i \mid \omega_i) = 1$ and $P(x_i \mid \omega_i) = q < 1$ where $q > 1 - p \geq \frac{1}{2}$. The prior weakly disfavors the plaintiff, that is, $p \leq \frac{1}{2}$. The assumption implies $P(\omega_1 \mid x_1) > \frac{1}{2}$ and $P(\omega_1 \mid x_0) < \frac{1}{2}$. For simplicity, y perfectly reveals the true fact. Thus, y can overturn the posterior based on x alone.

Investigation is followed by a communication phase in which the parties may report to the arbitrator. Reports are denoted by r_j, $j \in \{P, D\}$. If party j was unsuccessful in obtaining the evidence, its submission is by force the empty report $r_j = \emptyset$. If the party was successful and the state of the evidence is S, its report belongs to the set $\{\emptyset, (x, \emptyset)\}$ where $\emptyset$ means that the party submits nothing and $(x, \emptyset)$ that it reports only x. If the party was successful and the state of the evidence is M, its report belongs to the set $\{\emptyset, (x, \emptyset), (\emptyset, y), (x, y)\}$. When a party reports nothing, the arbitrator does not know whether the party was truly unsuccessful or whether it chose not to submit the evidence. When a party reports $(x, \emptyset)$ and the opponent is silent, the arbitrator does not know whether the disclosing party could also have submitted y. When a party reports $(\emptyset, y)$ or (x, y), however, the state of the evidence is revealed and the fact at issue is revealed as well because y is perfectly informative.

The arbitrator's updated belief that $\omega = \omega_1$, denoted μ, depends on the parties' reports. Her sequentially rational decision is then $d = 1$ if her belief $\mu > \frac{1}{2}$ and $d = 0$ if $\mu < \frac{1}{2}$. When $\mu = \frac{1}{2}$, the arbitrator is indifferent. As tie-breaker, we impose that she then finds for the defendant. Thus, P prevails only if the arbitrator believes that ω_1 is strictly more likely than not.

The foregoing describes a procedure without the opportunity of cross-examination, which we take as benchmark. The timeline is then as follows. First, nature chooses the true fact, whether the evidence consists of one or two pieces, and the realizations of the pieces of evidence. Next, the parties simultaneously choose their investigation efforts, e_P and e_D respectively, and Nature chooses whether they access the evidence or not, all of which is private information. At the subsequent stage, the parties simultaneously submit their reports r_P and r_D. Next, the arbitrator observes the reports, updates her beliefs, and adjudicates. The solution concept is Perfect Bayesian Equilibrium.

2.2. Testing reports: cross and re-examination

After the presentation of evidence, a common law trial typically allows the opportunity of cross-examination and redirect examination. During cross-examination, the opposing counsel challenges the submitted evidence by testing its reliability and consistency. Re-examination, conducted by the counsel of the presenting party, addresses ambiguities or attempts to restore the credibility of the party's report. Cross and re-examination introduce two additional stages between the disclosure of evidence and adjudication; see Figure 1.

In our setting, cross-examination, and similarly for re-examination, provides information distinct from the direct evidence described so far because it targets not the true fact itself but whether a party withheld evidence. When a party discloses the evidence x and the other party is silent, the arbitrator forms a belief about the potential concealment of evidence by the disclosing party. A cross-examiner aims to influence this belief by eliciting information suggesting that the disclosing party withheld evidence, which would imply that the concealed evidence favored the opponent.[4]

Re-examination by the disclosing party's counsel, often described as a repair job, seeks to mitigate the unfavorable inferences. For instance, following a string of so-called leading questions, the cross-examined may be led to self-contradict, or the outcome may suggest inconsistencies. A cross-examiner will not ask the "one question too many" so as to prevent the cross-examined to fully explaining his answers. The re-examiner may ask that question. At the adjudication stage, the arbitrator's belief about the true fact will depend not only on the initial reports but also on the outcomes of both cross-examination and re-examination.

2.2.1 Test space

Information about the possible withholding of evidence, when a party only disclosed x, amounts to information about whether the state of the evidence is M or S. We make two basic assumptions. First, examiners are "constrained" information designers, meaning that there are limits to what can be uncovered through cross and re-examination. Second, the cross and re-examiner face the same constraints and, in particular, the outcome of cross and re-examination is "connected," to use the terminology of Gentzkow and Kamenica (2017a, 2017b); see below.[5]

To capture the informational constraints, we assume that an infinite sequence of questions would at most reveal the realization z of a signal defined by the conditional densities $f_M(z)$ and $f_S(z)$, with the interval $Z = [\underline{z}, \bar{z}]$ as support (the bounds need not be finite) and with likelihood ratio $f_M(z)/f_S(z)$ strictly monotonic in z. A *finite* sequence of questions yields some coarser signal $\chi(z) \in K$, where K is a finite index set. The conditional probabilities are

$$\Pr(\chi = k \mid j) = \int_{\pi(k)} f_j(z) dz, \quad j \in \{M, S\}, \ \ k \in \mathrm{K}, \tag{1}$$

where $\pi(k) = \{z : \chi(z) = k\}$ and therefore $\{\pi(k)\}_{k \in K}$ is a partition of Z. The set of possible examination tests is the space of such signals over all finite partitions.[6]

An interpretation is as follows. The examined party is assumed to observe the realization z and is subject to yes–no questions of the form "is z in A?" where A is a subset of Z.

[4] We assume that there is no penalty for withholding evidence. Legal advice is routinely about the selection of information to disclose to a tribunal (Kaplow and Shavell 1989).

[5] Gentzkow and Kamenica (2017b) show that the Blackwell-connectedness condition is necessary and sufficient for competition in persuasion to increase information revelation.

[6] This is similar to the partition representation of signals described in Green and Stokey (1978).

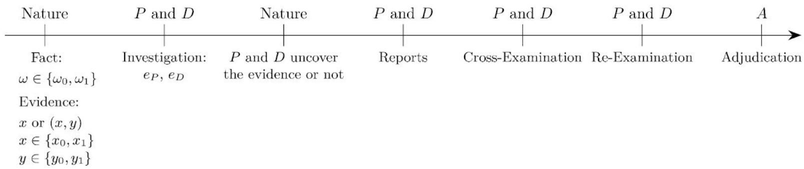

Figure 1. Time line.

Questions are answered truthfully. A positive answer to "is z in A?" reveals that $z \in A$, a negative answer that $z \in Z\, A$. After the first question, the examiner may ask another yes–no question, yielding a finer partition of Z, and so on. Figure 2 illustrates an examination strategy involving two questions. The first question is "is z in $[\underline{z}, z^*]$?," see the top line of Figure 2; if the answer is no, there is no further question; if the answer is yes, the examiner asks "is z in $[\underline{z}, z']$?" where $z' < z^*$, see the middle line in the figure. The outcome is the ternary partition at the bottom of the figure. The outcome would also be the same if asking the second question is not contingent on the answer to the first questions; indeed, the order of the questions is immaterial.

It could be that the first question is asked by the cross-examiner and the second question by the re-examiner. This formulation ensures that the tests conducted consecutively by a cross-examiner and a re-examiner are "connected," that is, the re-examiner refines the information obtained through cross-examination. We provide the intuition for the role that this will play. Let $f_M(z)/f_S(z)$ be decreasing in z. Large values of z are then more indicative that the state of the evidence is M. Suppose the cross-examiner asks, "is z in $[\underline{z}, z^*]$?," yielding the binary signal $\chi(z) = b$ if $z \leq z^*$ and $\chi(z) = g$ otherwise, as shown at the top of Figure 2. The likelihood ratios of M versus S, depending on the realizations of the binary signal, then satisfy

$$\frac{\Pr(\chi = b \mid M)}{\Pr(\chi = b \mid S)} > 1 > \frac{\Pr(\chi = g \mid M)}{\Pr(\chi = g \mid S)}.$$

Thus, the realization $\chi = b$ is "bad news" for the cross-examined party, in the sense that the adjudicator will revise upwards her belief that the cross-examined concealed evidence. Suppose that M is then sufficiently likely for the adjudicator to rule against the cross-examined party. Being interested, the examiner will of course have strategically chosen the scope of the question (as defined by z^*) to make the probability of "bad news" as large as possible.

When cross-examination turns bad, the counsel of the cross-examined party may conduct a re-examination. The purpose is counterpersuasion. Consider a re-examination consisting of the question "is z in $[\underline{z}, z']$?," yielding the binary signal with realizations in $\{b', g'\}$. The joint signal from cross and re-examination then yields the partition identified by b', bg', and g at the bottom of Figure 2. The likelihood ratios will satisfy

$$\frac{\Pr(\chi = b' \mid M)}{\Pr(\chi = b' \mid S)} > \frac{\Pr(\chi = b \mid M)}{\Pr(\chi = b \mid S)} > \frac{\Pr(\chi = bg' \mid M)}{\Pr(\chi = bg' \mid S)}.$$

When the outcome is bg', it may then be that the state of the evidence M is not sufficiently likely for the adjudicator to rule against the cross-examined. By refining the information

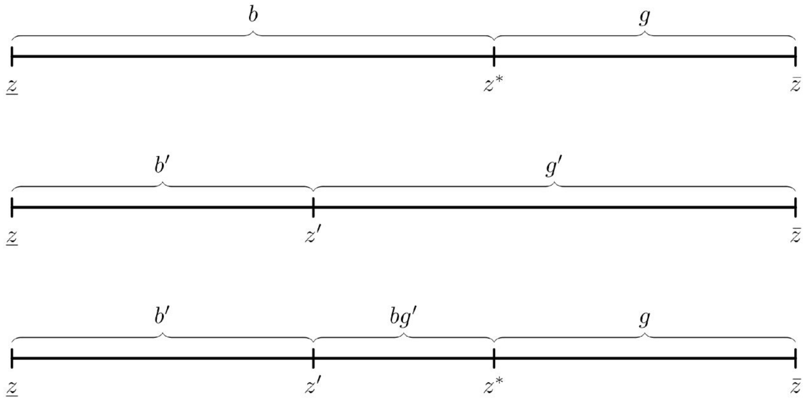

Figure 2. Two yes–no questions.

generated by cross-examination, re-examination will then have succeeded in reducing the probability that the examined party fails examination; failure will now occur only when $\chi(z) = b'$.

2.2.2 *Informativeness*

Examination impacts the arbitrator's decision only if it can be sufficiently informative. This depends on the densities $\{f_S, f_M\}$ defining the underlying signal on which the cross and re-examination tests are built. Because the examined party either withheld evidence or did not, the informativeness of the underlying signal is completely characterized by its performance in the design of binary tests. A binary test is a signal $\chi(z) \in \{b, g\}$ and an associated partition $\{\pi(b), \pi(g)\}$ of Z. Let

$$\alpha = \Pr(\chi = b \mid S) = \int_{\pi(b)} f_S(z) dz, \tag{2}$$

$$\beta = \Pr(\chi = b \mid M) = \int_{\pi(b)} f_M(z) dz. \tag{3}$$

Different partitions of Z generate different pairs (α, β). A binary test is efficient at level α if it yields the largest feasible β for the given α. From well-known results on tests of hypotheses, efficiency requires that $\chi(z) = b$ if $f_M(z)/f_S(z)$ is greater than some critical value that depends on α, otherwise $\chi(z) = g$. For instance, the two binary tests illustrated in Figure 2 are efficient but differ by the level of α.

We denote by $\widehat{\beta}(\alpha)$ the largest feasible β for a given α. The so-called power function $\widehat{\beta}(\alpha)$ is increasing and strictly concave, with $\widehat{\beta}(0) = 0$, $\widehat{\beta}(1) = 1$, and $\widehat{\beta}(\alpha) > \alpha$ for $\alpha \in (0, 1)$.[7] An underlying signal is Blackwell-more informative than another if its associated power function is greater than that of the other signal (Blackwell and Girschick 1954). Figure 3 depicts a family of $\widehat{\beta}(\alpha)$ curves ranked by the informativeness of the underlying

[7] For completeness, Lemma A1 in the Appendix provides a proof for the present setting. For a more general discussion, see Lehmann and Romano (2005), among others.

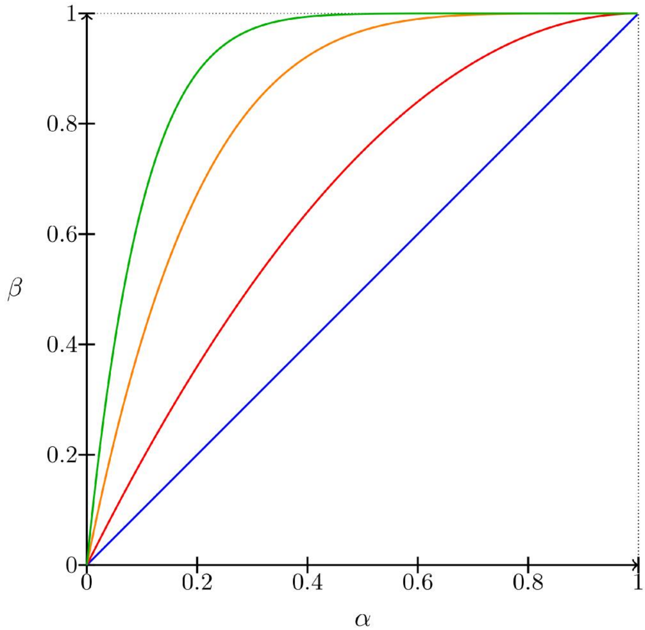

Figure 3: $\beta = \widehat{\beta}(\alpha)$ curves ordered by the Blackwell-informativeness of the underlying signal.

signal.[8] The straight line is for a completely uninformative signal with $f_M = f_S$. In the next sections, some results require the underlying signal to be sufficiently informative. By this, we mean a $\widehat{\beta}(\alpha)$ curve sufficiently above the diagonal in the (α, β) space; we will give a precise sufficient condition.

2.3. Procedures compared

We compare procedures with respect to the quality of adjudication, defined as the probability that the adjudicator makes the correct decision. The procedures are no opportunity of cross-examination, allowing only cross-examination, and cross-examination with the opportunity of redirect examination. Although a common law trial always involves both cross and re-examination, it is useful to consider the possibility of cross-examination on its own. First, in some legal systems or arbitration procedures, parties have more limited opportunities for redirect; for instance, the judge or arbitrator may intervene more actively at that stage. Second, by considering cross-examination on its own versus cross and re-examination, we will be able to demonstrate the important role of party-led or judge-led re-examination.

3. EQUILIBRIUM RESULTS

When successful in uncovering the evidence and when the state of the evidence is M, the plaintiff reports $y = y_1$ and the defendant reports $y = y_0$. Doing so cannot harm and may strictly benefit the party depending on the opponent's report and who bears the burden of proof.

[8] The curves are $\widehat{\beta}(\alpha) = 1 - (1-\alpha)^{\gamma}$ for $\gamma \geq 1$. The densities of the underlying signals are $f_M(z) = \gamma_M \exp(-\gamma_M)$ and $f_S(z) = \gamma_S \exp(-\gamma_S)$, with $\gamma_M / \gamma_S = \gamma$. Informativeness increases with γ.

Next, consider the pair of reports $r_P = (x_1, \emptyset)$ and $r_D = \emptyset$. This cannot arise when the state of the evidence is M and the defendant also uncovered the evidence: if $y = y_1$, the plaintiff would have reported it; if $y = y_0$, the defendant would have reported it. However, these reports may arise if the state of the evidence is S or if it is M together with $y = y_0$ and the defendant was unsuccessful in uncovering the evidence. The arbitrator weighs the two possibilities. A similar argument holds following $r_P = \emptyset$ and $r_D = (x_0, \emptyset)$. Applying Bayes' rule, the arbitrator's updated belief is

$$\mu((x_1, \emptyset), \emptyset) = \frac{(1-\theta)P(x_1, \omega_1)}{(1-\theta)\sum_i P(x_1, \omega_i) + \theta P(x_1, y_0, \omega_0)(1-e_D)}, \quad (4)$$

$$\mu(\emptyset, (x_0, \emptyset)) = \frac{(1-\theta)P(x_0, \omega_1) + \theta P(x_0, y_1, \omega_1)(1-e_P)}{(1-\theta)\sum_i P(x_0, \omega_i) + \theta P(x_0, y_1, \omega_1)(1-e_P)}, \quad (5)$$

where e_D and e_P are the arbitrator's conjectures of the parties' investigation efforts.

For e_P and e_D less than unity, as will always be the case, $\mu((x_1, \emptyset), \emptyset) < \frac{1}{2}$ if θ is sufficiently large, which goes against the plaintiff. By disclosing x_1, the plaintiff reveals that he has uncovered the evidence. Because the arbitrator takes into account the possibility that y_0 was withheld, the plaintiff will not prevail if it is very likely that the evidence consists of two pieces. Similarly, $\mu(\emptyset, (x_0, \emptyset)) > \frac{1}{2}$ if θ is sufficiently large, which goes against the defendant.[9] We introduce a condition ensuring that a party is credible when reporting an a priori favorable x and the opponent is silent.

Assumption 2: $\theta < \frac{p+q-1}{pq}$.

The assumption simplifies the exposition by limiting the number of possible equilibria.

Lemma 1. *For all e_P and e_D, $\mu((x_1, \emptyset), \emptyset) > \frac{1}{2}$ and $\mu(\emptyset, (x_0, \emptyset)) < \frac{1}{2}$.*

We first describe the equilibrium outcome without the opportunity of cross-examination.

3.1 Procedure without cross-examination

The following communication strategies are part of an equilibrium.[10] An informed plaintiff reports

$$r_P = \begin{cases} (x, y_1) \text{ if } M \text{ and } y_1, \\ (x_1, \emptyset) \text{ if } S \text{ and } x_1 \text{ or if } M \text{ and } (x_1, y_0), \\ \emptyset \text{ otherwise.} \end{cases} \quad (6)$$

An informed defendant reports

[9] To borrow from Bull and Watson (2019), a party's disclosure conveys information through the report's *face value* and as a signal of the party's private information.

[10] Some pair of reports will not arise at equilibrium, for instance $r_P = \emptyset$ and $r_D = (x_1, \emptyset)$. The arbitrator's beliefs are then obtained from completely mixed strategies where out-of-equilibrium moves are played with small probability. This yields the same decisions as would the raw posteriors.

$$r_D = \begin{cases} (x, y_0) \text{ if } M \text{ and } y_0, \\ (x_0, \emptyset) \text{ if } S \text{ and } x_0 \text{ or if } M \text{ and } (x_0, y_1), \\ \emptyset \text{ otherwise.} \end{cases} \tag{7}$$

Suppose that the arbitrator's decision is expected to be $d(\emptyset, \emptyset) = 0$, that is, P bears the burden of proof. At the investigation stage, the parties' expected payoffs are then

$$\bar{u}_P = e_P W(e_D) - C(e_P), \quad \bar{u}_D = -e_P W(e_D) - C(e_D), \tag{8}$$

where

$$W(e_D) = (1-\theta)P(x_1) + \theta[P(y_1) + P(x_1, y_0)(1 - e_D)] \tag{9}$$

is the plaintiff's expected gain (or probability of prevailing) conditional on having access to the evidence. The parties' expected payoffs depend on their investigation efforts and their conjecture of the opponent's effort. Because the plaintiff bears the burden of proof, he prevails only when submitting x_1 and the defendant is silent, or when submitting (x, y_1). Observe that $\theta P(x_1, y_0)(1 - e_D)$ is the probability that the plaintiff prevails with the deceitful report $r_P = (x_1, \emptyset)$ when the actual evidence is (x_1, y_0). This occurs when the defendant was unsuccessful in uncovering the evidence.

At equilibrium, e_P and e_D are best responses to one another and solve:

$$C'(e_P) = (1-\theta)P(x_1) + \theta[P(y_1) + P(x_1, y_0)(1 - e_D)] \tag{10}$$
$$C'(e_D) = e_P \theta P(x_1, y_0) \tag{11}$$

The solution is denoted e_P^{nc} and e_D^{nc} where *nc* stands for *no cross*.

Proposition 1 (No Cross-Examination).

An equilibrium with $\mu(\emptyset, \emptyset) < \frac{1}{2}$ and therefore with P bearing the burden of proof always exists, with a unique pair of investigation efforts satisfying $0 < e_D^{nc} < e_P^{nc} < 1$. An informed plaintiff prevails by submitting x_1 or (x, y_1); when he can, the defendant submits y_0 to counter the plaintiff.

The plaintiff bears the burden of proof in the sense that the arbitrator rules against the plaintiff when no evidence is submitted.[11] Both parties have an incentive to search for the evidence. P investigates because he bears the burden of proof; D investigates in order to possibly counteract the submission of x_1 by the plaintiff. However, the plaintiff invests more in uncovering the evidence and is therefore more likely to possess evidence, implying the arbitrator's belief $\mu(\emptyset, \emptyset) < \frac{1}{2}$ as shown in the proof. Figure 4 provides an example of the best response functions defined by Equations (10) and (11), denoted $\widehat{e}_P(e_D)$ and $\widehat{e}_D(e_P)$, respectively.[12]

[11] When the prior p is close to $\frac{1}{2}$, there is also an equilibrium where D bears the burden of proof. All of our results then hold after interchanging the role of P and D.

[12] In a contest model, Katz (1988) obtains that the expenditure of the ex ante favored party is "deterring" while that of the underdog is "provocative" (see also Daughety and Reinganum 2000). We get a similar result with the underdog as the party bearing the burden of proof.

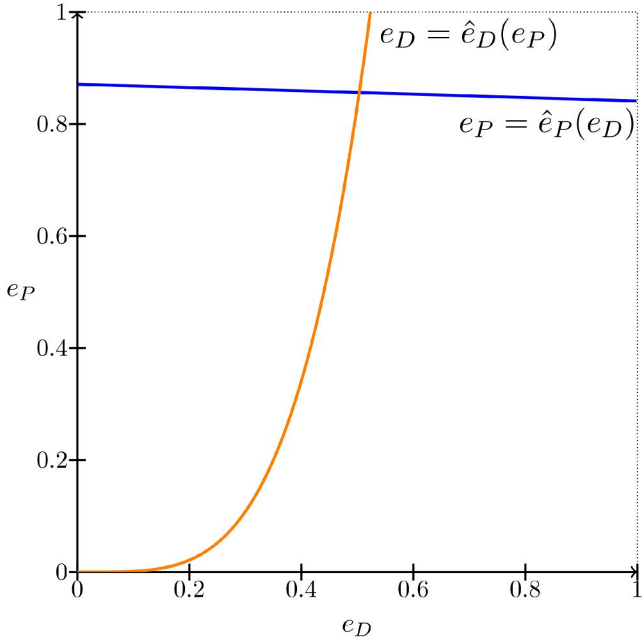

Figure 4: Investigation efforts ($p = .5$, $q = .7$, $\theta = .5$, $C(e) = .2e^5$, $e_P^{nc} = .856$, $e_D^{nc} = .503$).

When he would report $(x_0, \emptyset)$ according to the strategy defined in Equation (7), the defendant is in fact indifferent between that report and remaining silent. It does not matter because either the defendant prevails when the plaintiff reports nothing (the burden of proof being on the plaintiff) or the plaintiff prevails by submitting the overpowering (x_0, y_1). Proposition 1 therefore also holds when the defendant's communication strategy is

$$r_D = \begin{cases} (x, y_0) \text{ if } M \text{ and } y_0, \\ \emptyset \text{ otherwise.} \end{cases} \tag{12}$$

The investigation efforts are the same because the parties' expected payoffs in Equations (8) and (9) are unaffected.[13]

3.2 Cross-examination

The procedure now allows a party submitting evidence to be cross-examined by the opponent's counsel. This can play a role when a party reports $(x, \emptyset)$ and the opponent does not submit counterevidence. Because the defendant runs the risk of failing the cross-examination test when reporting $(x_0, \emptyset)$, that is, it is believed that evidence was withheld, the defendant is now better off with the communication strategy defined in Equation (12). Thus, when cross-examination is permitted, the defendant only submits the overpowering y_0 if he can and otherwise remains silent. By contrast, because he bears the burden of proof, the plaintiff's communication strategy is still Equation (6). Even though he may be cross-examined when reporting $(x_1, \emptyset)$, there is a chance that he will pass the test; the latter is ensured by the fact that cross-examination cannot be perfectly revealing.

[13] When no evidence is submitted, the arbitrator's belief is impacted but it still satisfies $\mu(\emptyset, \emptyset) < \frac{1}{2}$.

We observe that the arbitrator's skepticism vis-à-vis the plaintiff, following the reports $r_P = (x_1, \emptyset)$ and $r_D = \emptyset$, depends on her conjecture about the defendant's investigation effort. Indeed, we have that $\mu((x_1, \emptyset), \emptyset) = (1 - \nu(e_D))P(\omega_1 \mid x_1)$ where

$$\nu(e_D) = \frac{\theta P(x_1, y_0)(1 - e_D)}{(1 - \theta)P(x_1) + \theta P(x_1, y_0)(1 - e_D)} \tag{13}$$

is the arbitrator's belief that the plaintiff withheld evidence. The raw posterior $P(\omega_1 \mid x_1)$ is discounted by $\nu(e_D)$, which is decreasing in the conjecture of the defendant's investigation effort. The greater the probability that the defendant uncovered the evidence, the more likely it is that there is a single piece of evidence when the defendant is silent; hence, that the plaintiff did not conceal evidence.

Following cross-examination, the arbitrator's belief that the plaintiff withheld evidence is $\nu(e_D, \chi)$, as it will also depend on the outcome $\chi \in \{b, g\}$ of the test. Her belief that the true fact is ω_1 is now

$$\mu((x_1, \emptyset), \emptyset, \chi) = [1 - \nu(e_D, \chi)]P(\omega_1 \mid x_1),$$

with

$$\nu(e_D, \chi = b) = \frac{\theta P(x_1, y_0)(1 - e_D)\beta}{(1 - \theta)P(x_1)\alpha + \theta P(x_1, y_0)(1 - e_D)\beta},$$

where β is the probability of the cross-examination outcome $\chi = b$ when the plaintiff concealed evidence, α is the probability of the same outcome when the plaintiff was truthful.

Lemma 2. *$\mu((x_1, \emptyset), \emptyset, \chi = b) \leq \frac{1}{2}$ is equivalent to*

$$\alpha k_P(1 - \theta) \leq \beta k_D \theta(1 - e_D), \tag{14}$$

where $k_P := P(x_1, \omega_1) - P(x_1, \omega_0)$ and $k_D := P(x_1, \omega_0)$.

> Cross-examination is effective only when the test satisfies condition (Equation (14)). We note that Assumption 2 can be rewritten as $k_P(1 - \theta) > k_D\theta$.[14] Therefore, Equation (14) holds (with positive probability) only if β is sufficiently larger than α. The outcome $\chi = b$ is then the bad outcome from the plaintiff's point of view. It follows that $\chi = g$ is the good outcome with $\mu((x_1, \emptyset), \emptyset, \chi = g) > \frac{1}{2}$. We will refer to α and β as the false and true positive rates respectively, where a positive outcome means that the arbitrator is sufficiently convinced that the plaintiff withheld evidence.

Whether there are cross-examination strategies satisfying the condition in Lemma 2 depends on whether cross-examination tests have the potential to be sufficiently informative, compared to the information revealed by the reports (or lack thereof). We assume the following.

Assumption 3: $\alpha k_P(1 - \theta) < k_D\theta(\beta - e_D^{nc})$ for some $\beta \leq \widehat{\beta}(\alpha)$, $\alpha \in (0, 1)$.

The assumption is satisfied if the $\widehat{\beta}(\alpha)$ curve is sufficiently above the positive diagonal in the (α, β) space, that is, the underlying signal is sufficiently informative in the sense of

[14] The inequality in the assumption is $\theta < k_P/(k_P + k_D) = (p + q - 1)/(pq)$.

Blackwell (see Figure 3 and its explanations for further details). Observe that the assumption also implies that there exists cross-examination tests with α and β such that

$$\alpha k_P(1-\theta) < \beta k_D \theta(1-e_D^{nc}), \tag{15}$$

that is, the equilibrium without the opportunity of cross-examination is overturned when cross-examination is permitted. The investigation efforts e_P^{nc} and e_D^{nc} are then no longer part of an equilibrium because the plaintiff now faces the threat of effective cross-examination.[15]

3.2.1 *Investigation efforts*

Given an effective cross-examination strategy, the probability that the plaintiff prevails conditional on uncovering the evidence is now

$$W(e_D,\alpha,\beta) = (1-\theta)P(x_1)(1-\alpha) \tag{16}$$
$$+\theta[P(y_1)+P(x_1,y_0)(1-e_D)(1-\beta)].$$

When the state of the evidence is S, the plaintiff does not always prevail when reporting $r_P=(x_1,\emptyset)$ because of the false positive rate α. When the state of the evidence is M and the plaintiff deceitfully reports $r_P=(x_1,\emptyset)$, the plaintiff may be countered either by the defendant reporting $r_P=(x_1,y_0)$ or, when the defendant is unable to do so, by cross-examination with true positive rate β.

The investigation efforts therefore solve:

$$C'(e_P) = (1-\theta)P(x_1)(1-\alpha) \tag{17}$$
$$+\theta[P(y_1)+P(x_1,y_0)(1-e_D)(1-\beta)]$$
$$C'(e_D) = e_P\theta(1-\beta)P(x_1,y_0). \tag{18}$$

The solution to Equations (17) and (18) yields the equilibrium investigation efforts e_P^{ce} and e_D^{ce}, where *ce* stands for *cross-examination*, assuming (α,β) is the equilibrium cross-examination strategy.

3.2.2 *Cross-examination strategy*

The reports $r_P=(x_1,\emptyset)$ and $r_D=\emptyset$ can arise either (i) when the defendant did not access the evidence or (ii) when the defendant accessed the evidence and the state of the evidence is S, implying that the plaintiff did not in fact withhold evidence. The cross-examiner (i.e., the defendant's counsel) may therefore be of two types: either uninformed of the state of the evidence or informed. The counsel is uninformed when the defendant did not access the evidence (and possibly also if that information is not shared by the defendant because it was unfavorable). When informed, the counsel knows that the evidence reduces to the single piece disclosed by the plaintiff. In our baseline setting, we assume that the counsel then nevertheless cross-examines the plaintiff in order to induce the adjudicator to believe that

[15] Note that, for a given $\widehat{\beta}(\alpha)$ curve, Assumption 3 (and similarly for inequality [15]) is easier to satisfy when e_D^{nc} is not too large. When e_D^{nc} is large, the defendant is most likely informed of the state of the evidence. The reports $r_P=(x_1,\emptyset)$ and $r_D=\emptyset$ then suggest that the state of the evidence is S, otherwise the defendant would most probably have disclosed y_1. To counter the belief favoring the plaintiff requires an adequately informative cross-examination test.

evidence was withheld.[16] In Appendix B, we consider an honest cross-examiner who does not attempt to mislead in this manner. We show that our results concerning the effect of cross and redirect examination are unchanged.

An informed (and dishonest) cross-examiner raises the possibility that a cross-examination strategy could reveal the cross-examiner's information status. In a setting involving an informed persuader with two possible types, Perez-Richet (2014) shows that one can confine attention to pooling equilibria where both types choose the persuasion strategy that is optimal for the "high type," defined as the one who would want his type to be revealed. An informed cross-examiner does not want his type to be revealed, as this would reveal that the plaintiff did not suppress evidence, thereby defeating the purpose of cross-examination. We therefore consider the cross-examination strategy that an uninformed cross-examiner would choose.

Before questioning the plaintiff, the uninformed cross-examiner's belief about the state of the evidence is the same as the arbitrator's. The cross-examiner seeks to maximize the probability that the plaintiff fails the test, subject to α and β satisfying the inference constraint, Equation (14). The cross-examination test therefore solves

$$\max_{\alpha,\beta}(1-\nu(e_D^{ce}))\alpha+\nu(e_D^{ce})\beta$$

s.t.

$$\alpha k_P(1-\theta)\leq\beta k_D\theta(1-e_D^{ce}),\ \ \beta\leq\widehat{\beta}(\alpha),\ \ \alpha\in[0,1]. \tag{19}$$

Both constraints are binding: the solution is $\beta=\widehat{\beta}(\alpha)$ where $\alpha>0$ solves

$$\frac{\widehat{\beta}(\alpha)}{\alpha}=\frac{k_P(1-\theta)}{k_D\theta(1-e_D^{ce})}. \tag{20}$$

Per Assumption 3, that equation always has a solution $\alpha\in(0,1)$ if the defendant's investigation effort is less than e_D^{nc} at equilibrium, which will be the case. The solution is unique because, by concavity of $\widehat{\beta}$, the ratio $\widehat{\beta}(\alpha)/\alpha$ is strictly decreasing. Note that, per Lemma 2, a binding inference constraint implies $\mu((x_1,\emptyset),\emptyset,\chi=b)=\frac{1}{2}$.

In Figure 5, the straight line with slope determined by e_D^{nc} intersects the $\widehat{\beta}(\alpha)$ curve, that is, the inequality (Equation 15) can be satisfied. The slope of the other line is determined by the defendant's investigation effort e_D^{ce} in the equilibrium with cross-examination. The intersection of this line with the $\widehat{\beta}(\alpha)$ curve yields the cross-examination strategy with false-positive rate α^{ce} and true positive rate $\widehat{\beta}(\alpha^{ce})$.

Proposition 2 (Cross-examination).

D reports (x,y_0) when he can and otherwise cross-examines P when the latter reported $(x_1,\emptyset)$. D's cross-examination strategy is $\beta^{ce}=\widehat{\beta}(\alpha^{ce})$ where $\alpha^{ce}>0$ solves (20). The parties' investigation efforts satisfy $e_D^{ce}<e_P^{ce}$, with $e_P^{ce}<e_P^{nc}$ and $e_D^{ce}<e_D^{nc}$.

The defendant investigates less because cross-examining the plaintiff is a substitute to uncovering countervailing evidence. Although the reduction in the defendant's investigation effort makes it less likely that the defendant will produce

[16] We thank a referee for pointing out that such a conduct would be considered unethical and possibly illegal.

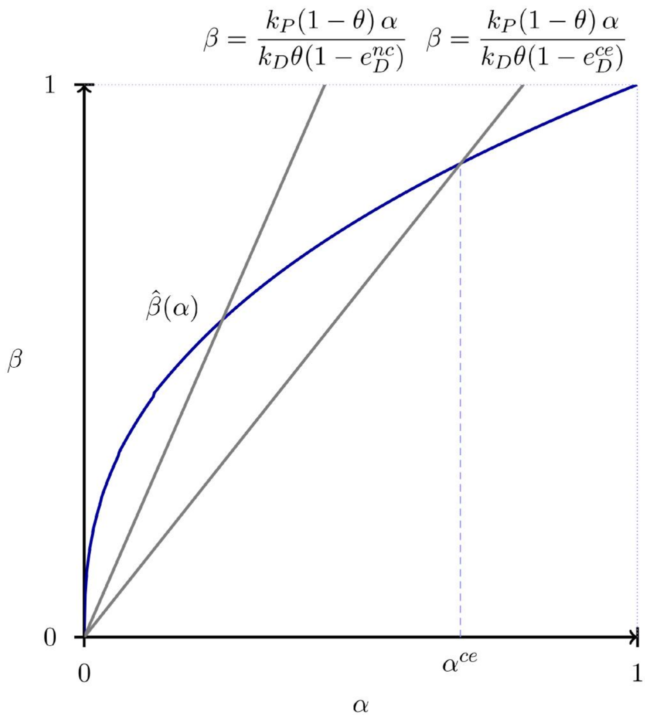

Figure 5: Cross-examination strategy.

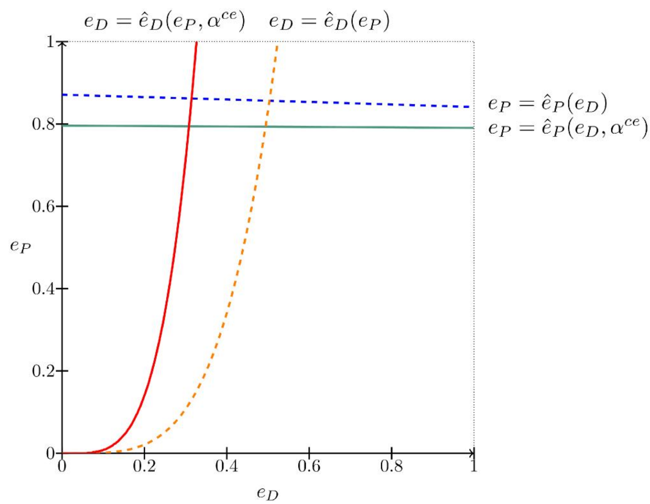

Figure 6: Investigation efforts with cross-examination ($p = .5$, $q = .7$, $\theta = .5$, $C(e) = .2e^5$; $f_j(z) = \gamma_j \exp(-\gamma_j)$, $j = M, S$; $\gamma_S = 5\gamma_M$; $\alpha^{ce} = .440$, $\beta^{ce} = .849$, $e_P^{ce} = .794$, $e_D^{ce} = .308$).

counterevidence, the plaintiff also investigates less because of the threat of cross-examination with sufficiently large true and false positive rates.

Figure 6 illustrates the effect of allowing cross-examination on the parties' investigation efforts. The setting is the same as in Figure 4. The dashed curves are the previous best response functions when cross-examination is not allowed. The thick curves are the ones when cross-examination is allowed, computed at the equilibrium cross-examination strategy.[17] The defendant investigates significantly less, and the plaintiff slightly less.

3.3 Re-examination

Following the reports $r_P = (x_1, \emptyset)$ and $r_D = \emptyset$, the relevant stages are now (a) the defendant conducts a cross-examination test; (b) the plaintiff's counsel observes its outcome and, if it is unfavorable, conducts a re-examination test; (c) the arbitrator observes the cross and re-examination tests, their outcome, and adjudicates. As described in (b), the re-examiner observes the outcome of cross-examination before choosing his own re-examination strategy. Li and Norman (2021) show that, when signals are connected, it is without loss of generality to analyze sequential persuasion as if signal realizations were observed only after all players have chosen their strategies. It is also without loss to focus on *one-step* equilibria, where nontrivial information is generated only by the first sender's strategy. In our setting, a one-step equilibrium is a situation where the re-examiner has no incentives to refine the test conducted by the cross-examiner.

Thus, the cross-examiner designs a test that maximizes the probability of disfavoring the plaintiff, subject to the constraint that the test will not be defeated. In what follows, we write $\alpha = \int_Z \psi(z) f_S(z) dz$ and $\beta = \int_Z \psi(z) f_M(z) dz$ where $\psi : Z \to \{0,1\}$. Denoting by e_D^{cr} the arbitrator's conjecture of the defendant's investigation effort (*cr* stands for cross and re-examination), the cross-examiner solves

$$\max_{\psi(z)\in\{0,1\}} (1-\nu(e_D^{cr})) \int_Z \psi(z) f_S(z) dz + \nu(e_D^{cr}) \int_Z \psi(z) f_M(z) dz \tag{21}$$

subject to the inference constraint (Equation (14)), rewritten as

$$k_P(1-\theta) \int_Z \psi(z) f_S(z) dz \le k_D \theta (1-e_D^{cr}) \int_Z \psi(z) f_M(z) dz, \tag{22}$$

and to the additional constraint

$$\begin{aligned} &k_P(1-\theta) \textstyle\int_{z'}^{z''} \psi(z) f_S(z) dz \\ &\le k_D \theta (1-e_D^{cr}) \textstyle\int_{z'}^{z''} \psi(z) f_M(z) dz, \quad \text{for all } z'' > z' \text{ in } Z. \end{aligned} \tag{23}$$

If Equation (23) was not satisfied over some interval (z', z''), the re-examiner would use a test that reveals realizations in this interval, thereby partly defeating the cross-examiner's test.

Lemma 3. *In a one-step equilibrium under cross and re-examination, D's cross-examination strategy maximizes $\beta(\alpha) k_D \theta (1-e_D^{cr}) - \alpha k_P (1-\theta)$ with respect to α.*

The above maximization problem has an interior solution, so that α solves

[17] The underlying signal is defined by exponential densities; see the specification in the figure.

$$\widehat{\beta}'(\alpha) = \frac{k_P(1-\theta)}{k_D\theta(1-e_D^{cr})}. \tag{24}$$

Proposition 3. ***(Cross and re-examination)*** *The parties' investigation efforts satisfy $e_D^{cr} < e_P^{cr}$ with $e_D^{ce} < e_D^{cr} < e_D^{nc}$ and $e_P^{cr} < e_P^{nc}$. For the arbitrator, the relevant information from examination is the binary signal with $\beta^{cr} = \widehat{\beta}(\alpha^{cr})$ where $\alpha^{cr} < \alpha^{ce}$ and solves (Equation (24)).*

Figure 7 compares the examination outcome when only cross-examination is permitted and when re-examination is also allowed. Because $e_D^{cr} > e_D^{ce}$,

$$\widehat{\beta}'(\alpha^{cr}) = \frac{k_P(1-\theta)}{k_D\theta(1-e_D^{cr})} > \frac{k_P(1-\theta)}{k_D\theta(1-e_D^{ce})} = \frac{\widehat{\beta}(\alpha^{ce})}{\alpha^{ce}}.$$

The false-positive rate is smaller than with cross-examination alone and the true-positive rate is therefore also smaller.

3.4 Quality of adjudication

The probability of correct adjudication, equivalently the arbitrator's expected utility is

$$\bar{u}_A = p\Pr(d=1 \mid \omega_1) + (1-p)\Pr(d=0 \mid \omega_0) \tag{25}$$

where $\Pr(d \mid \omega)$ is the probability of decision d at equilibrium, conditional on the true fact being ω. Different procedures differ in the quality of adjudication through these conditional probabilities. We first rewrite the above in terms of the parties' investigation efforts and the outcome of examination.

Lemma 4. $\bar{u}_A = 1 - p + e_P\Lambda(e_D, \alpha, \beta)$ *where*

$$\Lambda(e_D, \alpha, \beta) := p\theta + [k_P(1-\theta)(1-\alpha) - k_D\theta(1-e_D)(1-\beta)].$$

If the parties never communicated evidence, the arbitrator would always find for the defendant. The probability of correct adjudication would then be equal to $1-p$. The term $e_P\Lambda$ in the lemma is the value added by the investigation and communication phases in possibly overturning the arbitrator's default decision. We will refer to Λ as the value of communication because it is the increase in the probability of a correct decision conditional on the plaintiff's uncovering of the evidence and on the communication that ensues (including the possibility of silence). The value of communication depends on the evidence that the plaintiff could submit, the counterevidence that the defendant could submit, and the information potentially provided by cross and redirect examination.

When cross-examination is not allowed, α and β are equal to zero in the above expression. Substituting the defendant's equilibrium investigation effort, the value of communication is then

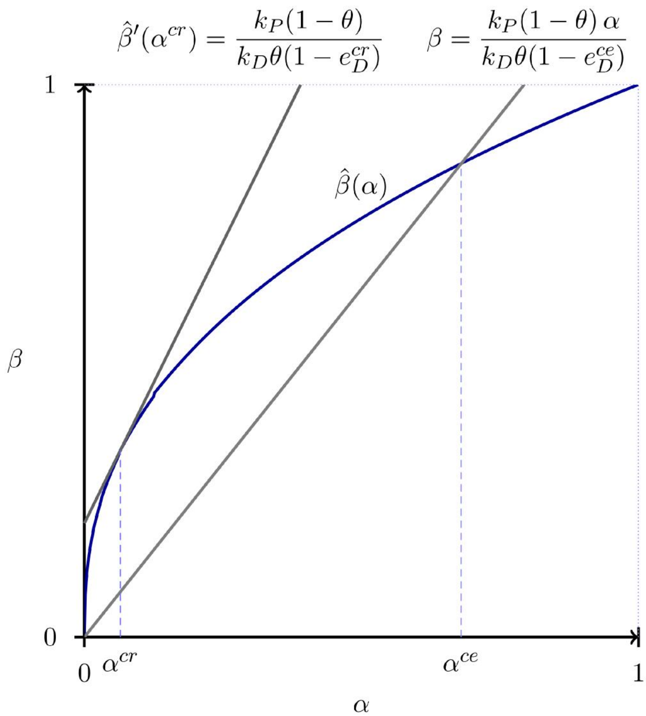

Figure 7: Cross and re-examination.

$$\Lambda^{nc} := \Lambda(e_D^{nc}, 0, 0) = p\theta + [k_P(1-\theta) - k_D\theta(1-e_D^{nc})]. \tag{26}$$

When cross-examination is permitted but without the possibility of redirect examination, the value of communication is

$$\Lambda^{ce} := \Lambda(e_D^{ce}, \alpha^{ce}, \beta^{ce}) = p\theta + [k_P(1-\theta)(1-\alpha^{ce}) - k_D\theta(1-e_D^{ce})(1-\beta^{ce})].$$

Substituting the equilibrium cross-examination strategy, that is, Equation (20), then yields

$$\Lambda^{ce} = p\theta + [k_P(1-\theta) - k_D\theta(1-e_D^{ce})]. \tag{27}$$

Between the two procedures, the value of communication differs only through the value of the defendant's investigation effort.

Proposition 4. *When only cross-examination is permitted without the possibility of re-examination, (i) the value of communication is reduced compared to no examination, $\Lambda^{ce} < \Lambda^{nc}$; (ii) the plaintiff investigates less, $e_P^{ce} < e_P^{nc}$, so that the quality of adjudication unambiguously deteriorates.*

The first claim follows from the fact that the defendant now investigates less, as shown in Proposition 2. Without this effect, the value of communication would remain the same: as in the example discussed in the introduction, the risk of error would merely shift from one kind (erroneously finding for the plaintiff) to the other kind (erroneously finding for the defendant). Because cross-examination yields sufficiently large false and true positive rates, the plaintiff has less incentives to investigate, despite the smaller probability that the defendant presents countervailing evidence.

When both cross and redirect examination are allowed, the value of communication is

$$\Lambda^{cr} := \Lambda(e_D^{cr}, \alpha^{cr}, \beta^{cr}) = p\theta + [k_P(1-\theta)(1-\alpha^{cr}) - k_D\theta(1-e_D^{cr})(1-\beta^{cr})].$$

From Lemma 3, it is easily seen that α^{cr} maximizes $\Lambda(e_D^{cr}, \alpha, \widehat{\beta}(\alpha))$, that is, the value of communication is maximized conditional on the defendant's investigation effort.[18] By itself, this improves the value of communication compared to no examination. However, the defendant now investigates less, $e_D^{cr} < e_D^{nc}$, which goes in the opposite direction. Part (i) of our next result shows that the first effect dominates with respect to the value of communication.

Proposition 5. *When both cross and redirect-examination are allowed, (i) the value of communication is increased compared to no examination, $\Lambda^{cr} > \Lambda^{nc}$; (ii) because the plaintiff investigates less, $e_P^{cr} < e_P^{nc}$, the effect on the quality of adjudication is ambiguous.*

The outcome following cross and re-examination is the same as if examination were conducted in a nonpartisan manner, for instance, by the adjudicator herself, assuming she has access to the same test space. Whether this is sufficient to improve the quality of adjudication then depends on the importance of the negative effect on the plaintiff's investigation effort.

Table 1 illustrates some of the possibilities. The potential evidence and the investigation costs are the same as in Figure 4. The test space I is the one used for Figure 6. Cross-examination alone then does significantly worse than no examination at all, but cross together with re-direct examination slightly improves the quality of adjudication. We also present the outcome under a different space of feasible examination tests, labeled test space II. Again, cross-examination alone does significantly worse than no examination at all. Now, however, even with cross and re-examination, the quality of adjudication deteriorates compared to no examination. To interpret the numerical results, note that the probability of correct adjudication would equal $(1-\theta)q + \theta = 0.85$ if the arbitrator had direct access to the evidence.

Figure 8 depicts the power functions for the test spaces I and II in our numerical example.[19] The equilibrium (α, β) pairs are shown by dots when only cross-examination is allowed and by triangles when both cross and redirect examination are permitted. In either procedure, under test space II compared to test space I, a deceitful party is detected more often (a larger β) and a truthful party more often appears to be a liar (a larger α). This is due to the "orientation" of the signal underlying the test spaces.[20] Loosely speaking, for test space II, signal realizations more often suggest that the plaintiff concealed evidence rather than not. The signal underlying state space I has the opposite orientation, that is, depending on the feasible tests proving one thing will be easier than proving the opposite. In our example, the defendant is better off under test space II. The defendant then investigates less but nevertheless more often prevails because of the larger false and true positive rates.

[18] This is an instance of the role of competition in improving information revelation. See Gentzkow and Kamenica (2017, 2017b).

[19] For the densities in Table 1, the power functions are $\widehat{\beta}_I(\alpha) = \alpha^{\frac{1}{5}}$ and $\widehat{\beta}_{II}(\alpha) = 1-(1-\alpha)^5$. The discriminatory power of dichotomic tests is often measured by the area under the power curve (AUC), here equal to 0.83 for both curves. This would generally be considered as relatively 'good' tests (Hosmer and Lemeshow 2000).

[20] See Fluet and Mungan (2025) for the definition of the 'orientation' of a signal.

Table 1. Procedures and examination sets.

	$\mathbf{e_P}$	$\mathbf{e_D}$	α	β	Λ	$\mathbf{1-p+e_P\Lambda}$
No examination	0.856	0.503	–	–	0.313	0.768
Test space I						
Cross only	0.794	0.308	0.440	0.849	0.298	0.737
Cross and redirect	0.844	0.412	0.048	0.545	0.325	0.774
Test space II						
Cross only	0.763	0.133	0.646	0.994	0.285	0.717
Cross and redirect	0.827	0.384	0.188	0.649	0.315	0.760

Note: $f_j(z) = \gamma_j \exp(-\gamma_j)$, $j = M, S$; test space I, $\gamma_S = 5\gamma_M$; test space II, $\gamma_M = 5\gamma_S$; $C(e) = e_i^5/5$; $\theta = .5$, $q = .7$, $p = .5$.

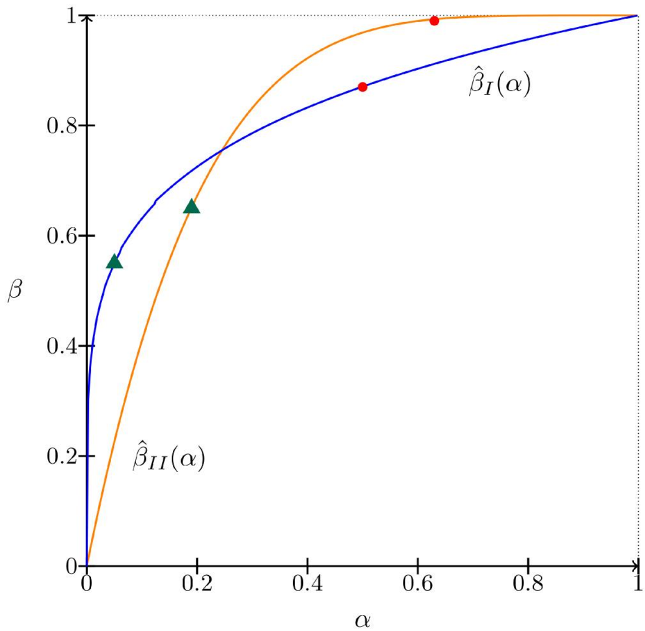

Figure 8: Differently oriented test spaces.

5. CONCLUDING REMARKS

Posner (1999: 1543) remarked that: "A principal social value of the right of cross-examination is deterrent: the threat of cross-examination deters some witnesses from testifying at all and others from giving false or misleading evidence." We studied a situation where cross-examination does indeed reduce the probability of misleading reports or testimonies. However, the quality of adjudication deteriorates if cross-examination is conducted without appropriate safeguards. On its own, the opportunity of cross-examination reduces incentives to acquire evidence both for the party facing the threat of cross-examination and for the one standing to benefit from it. A sophisticated Bayesian adjudicator would then make less informed inferences. These effects are mitigated when re-examination by one's own counsel is permitted. Re-examination disciplines cross-examination, yielding lower rates of false positives and improving incentives to acquire evidence for both parties, compared to cross-examination alone. Our analysis shows that the choice of procedure should be between either no examination at all or cross-examination together with re-examination or some substitute thereof, for instance, court-led examination.

Nevertheless, there is no guarantee that decision-making will improve compared to no examination at all. The threat of examination has a chilling effect on the gathering of evidence and incentives to come forward for the party with the burden of proof. That party stands to be less successful in manipulating the evidence. This bears a similarity with the issue of mandatory versus voluntary disclosure. As is well known, the ability to hide information (the "right to silence") is often an important part of the incentive to gather information. Mandatory disclosure may therefore be undesirable when information is costly. The trade-off is between how much is communicated conditional on the information acquired and how much information is acquired (Matthews and Postlewaite 1985; Farrell 1986; Shavell 1994; Wickelgren 2010; Schweizer 2017). Examination, whether cross or both cross and redirect, differs somewhat because its outcome is noisy. The threat of true positives acts like (stochastic) mandatory disclosure: it reduces incentives to acquire evidence but improves transparency. In contrast, the prospect of false positives reduces incentives to acquire evidence but garbles communication. There will be cases where the chilling effect is large compared to the improvement in communication.

As a final observation, the merits of examination have been discussed solely with respect to the quality of adjudication. It may be that society is concerned both with adjudication error and the parties' costs. The opportunity of examination typically reduces both parties' expenditures. Thus, there will be situations where adjudication is improved and costs are simultaneously reduced. Even when decision-making is not improved, society may still be better off depending on the weight accorded to procedural costs relative to adjudication error, as for instance in Sobel (1985) or Emons and Fluet (2009). However, the cost of mounting an efficient examination strategy may itself be nonnegligible, which should then also be factored in.

ACKNOWLEDGMENTS

For helpful comments on earlier versions of this study, we thank Jesse Bull, Andrew Daughety, Jeffrey Ely, Tim Friehe, Bruce Kobayashi, Erik Talley, Navin Kartik, Kathryn Spier, Joshua Teitelbaum, and Joel Watson. We are particularly grateful to Abraham Wickelgren, the Editor, and the three referees for their careful reading and suggestions.

Conflict of interest statement. None declared.

FUNDING

Funding support for this article was provided by the Social Sciences and Humanities Research Council of Canada (grant 435-2023-1671).

APPENDIX A

This appendix contains the proofs of the propositions and lemmas. We first introduce a lemma to prove a claim made in Section 2.2.

Lemma A1 $\widehat{\beta}(\alpha)$ is increasing and strictly concave, with $\widehat{\beta}(0) = 0$, $\widehat{\beta}(1) = 1$, and $\widehat{\beta}(\alpha) > \alpha$ for $\alpha \in (0, 1)$.

Proof. Let F_M and F_S denote the cdf's of f_M and f_S respectively. Let $\alpha = \int_Z \psi(z) f_S(z) dz$ and $\beta = \int_Z \psi(z) f_M(z) dz$ where $\psi : Z \to \{0, 1\}$. By Neyman-Pearson's lemma (see for instance Lehmann and Romano, 2005), β is maximized for a given α with $\psi(z)=1$ if $f_M(z)>kf_S(z)$ and $\psi(z)=0$ if $f_M(z)<kf_S(z)$, where k depends on α. Therefore, when $f_M(z)/f_S(z)$ is decreasing, $\widehat{\beta}(\alpha)=F_M(z(\alpha))$ where $z(\alpha)$ solves $F_S(z(\alpha))=\alpha$. When $f_M(z)/f_S(z)$ is increasing, $\widehat{\beta}(\alpha)=1-F_M(z(\alpha))$ where $z(\alpha)$ solves $1-F_S(z(\alpha))=\alpha$. In either case, $\widehat{\beta}'(\alpha)=f_M(z(\alpha))/f_S(z(\alpha))$ and is decreasing in α, implying that $\widehat{\beta}(\alpha)$ is strictly concave. That $\widehat{\beta}(\alpha)>\alpha$ for $\alpha\in(0,1)$ follows from the strict concavity, given that $\widehat{\beta}(1)=1$. ■

Proof of Lemma 1. From (4), $\mu((x_1,\emptyset),\emptyset)>\frac{1}{2}$ is equivalent to

$$(1-\theta)[P(x_1,\omega_1)-P(x_1,\omega_0)]>\theta P(x_1,y_0,\omega_0)(1-e_D).$$

Substituting from Assumption 1, a sufficient condition is

$$(1-\theta)(p+q-1)>\theta(1-q)(1-p),$$

which is equivalent to Assumption 2. Similarly, from Equation (5), a sufficient condition for $\mu(\emptyset,(x_0,\emptyset))<\frac{1}{2}$ is

$$\theta<\frac{q-p}{q(1-p)},$$

which is implied by Assumption 2. ■

Proof of Proposition 1. We first show that $0<e_D^{nc}<e_P^{nc}<1$; next, that $e_P^{nc}>e_D^{nc}$ implies $\mu(\emptyset,\emptyset)<\frac{1}{2}$.

(i) $0<e_D^{nc}<e_P^{nc}<1$

Rewrite (10) as $e_P=\widehat{e}_P(e_D)$ and (11) as $e_D=\widehat{e}_D(e_P)$. Because $C''>0$, $\widehat{e}'_P(e_D)<0$ and $\widehat{e}'_D(e_P)>0$. Because $C'(0)=0$ and $C'(1)\geq 1$, $\widehat{e}_P(e_D)\in(0,1)$ for all $e_D\in[0,1]$ and $\widehat{e}_D(e_P)<1$ for all $e_P\in[0,1]$. Let $\varphi(e_P)\equiv e_P-\widehat{e}_P[\widehat{e}_D(e_P)]$. Then $\varphi(0)\equiv-\widehat{e}_P[\widehat{e}_D(0)]<0$, $\varphi(1)\equiv 1-\widehat{e}_P[\widehat{e}_D(1)]>0$, and $\varphi'(e_P)=1-\widehat{e}'_P[\widehat{e}_D(e_P)]\widehat{e}'_D(e_P)>0$. Hence, there exists a unique $e_P\in(0,1)$ solving $\varphi(e_P)=0$. From (11), it follows that $e_D>0$.

To show that $e_D^{nc}<e_P^{nc}$, we note that the inequality is equivalent to $C'(e_P^{nc})>C'(e_D^{nc})$ and therefore to

$$(1-\theta)P(x_1)+\theta[P(y_1)+P(x_1,y_0)(1-e_D^{nc})]>e_P^{nc}\theta P(x_1,y_0).$$

The left-hand side is decreasing in e_D^{nc} and the right-hand side increasing in e_P^{nc}, so it suffices that the inequality holds at $e_P^{nc}=e_D^{nc}=1$, that is,

$$(1-\theta)P(x_1)+\theta[P(y_1)-P(x_1,y_0)]>0. \tag{A1}$$

The term in brackets satisfies

$$\begin{aligned}P(y_1)-P(x_1,y_0)&=P(\omega_1)-P(x_1,\omega_0)\\&=p-(1-p)(1-q)\\&=p(1-q)+p+q-1>0,\end{aligned}$$

which follows from Assumption 1.

(ii) $\mu(\emptyset,\emptyset)<\frac{1}{2}$

Applying Bayes' rule,

$$\mu(\emptyset,\emptyset)=\frac{p\Pr(r_P=\emptyset,r_D=\emptyset\mid\omega_1)}{\Pr(r_P=\emptyset,r_D=\emptyset)}$$

so that $\mu(\emptyset,\emptyset)<\frac{1}{2}$ if

$$\frac{\Pr(r_P=\emptyset,r_D=\emptyset\mid\omega_0)}{\Pr(r_P=\emptyset,r_D=\emptyset\mid\omega_1)}>\frac{p}{1-p}. \tag{A2}$$

The inequality (Equation (A2)) holds for all $p\le\frac{1}{2}$ if it does at $p=\frac{1}{2}$. Thus, it suffices that

$$\xi\equiv\Pr(r_P=\emptyset,r_D=\emptyset\mid\omega_0)-\Pr(r_P=\emptyset,r_D=\emptyset\mid\omega_1)>0.$$

Given the disclosure strategies,

$$\begin{aligned}\Pr(\,r_P=\emptyset,r_D=\emptyset\mid\omega_i\,)&=(1-e_P)(1-e_D)\\&+e_P(1-e_D)[(1-\theta)P(\,x_0\mid\omega_i\,)+\theta P(\,x_0,y_0\mid\omega_i\,)]\\&+(1-e_P)e_D[(1-\theta)P(\,x_1\mid\omega_i\,)+\theta P(\,x_1,y_1\mid\omega_i\,)]\end{aligned}$$

Substituting from Assumption 1, straightforward computations yield

$$\xi=(e_P-e_D)[2q-1+\theta(1-q)]>0,$$

where the sign follows from $q>1-p\ge\frac{1}{2}$ and $e_P>e_D$. ■

Proof of Lemma 2. From Bayes' rule, given that $\alpha=\Pr(\chi=b\mid S)$ and $\beta\equiv\Pr(\chi=b\mid M)$,

$$\mu((x_1,\emptyset),\emptyset,\chi=b)=\frac{[(1-\theta)P(x_1,\omega_1)\alpha+\theta P(x_1,y_0,\omega_1)(1-e_D)\beta}{\sum_i\{(1-\theta)P(x_1,\omega_i)\alpha+\theta P(x_1,y_0,\omega_i)(1-e_D)\beta\}}$$

Simplifying as in the proof of Lemma 1 yields Equation (14). ■

Proof of Proposition 2. To complete the discussion in the text, we first prove the existence of an equilibrium as described; next, we show that $e_D^{ce}<e_D^{nc}$ and conclude with the proof that $e_P^{ce}<e_P^{nc}$. The proof that $e_D^{ce}<e_P^{ce}$ is similar to the proof of $e_D^{nc}<e_P^{nc}$ in Proposition 1 and is omitted.

(i) Existence

Substitute for $\beta=\widehat{\beta}(\alpha)$ in Equations (17) and (18) and let $\bar{e}_P(\alpha)$ and $\bar{e}_D(\alpha)$ denote the solution given an arbitrary α. Using the same argument as in Proposition 1, such a solution exists and is unique. We show in (ii) below that $\bar{e}_D(\alpha)$ is strictly decreasing, with $\bar{e}_D(0)=e_D^{nc}$ because $\widehat{\beta}(0)=0$ and $\bar{e}_D(1)=0$ because $\widehat{\beta}(1)=1$.

Define

$$h(\alpha)\equiv 1-\frac{\alpha}{\widehat{\beta}(\alpha)}\frac{k_P(1-\theta)}{k_D\theta}. \tag{A3}$$

The function is strictly decreasing. Using l'Hôpital's rule,

$$h(0) = 1 - \frac{k_P(1-\theta)}{k_D\theta}\left(\frac{1}{\widehat{\beta}'(0^+)}\right) > 0$$

where the inequality follows from Assumption 3. Moreover,

$$h(1) = 1 - \frac{k_P(1-\theta)}{k_D\theta} < 0,$$

where the inequality follows from Assumption 2. Finally, Equations (17), (18), and (20) are simultaneously solved if $h(\alpha) = \bar{e}_D(\alpha)$. An equilibrium with $\alpha \in (0,1)$ therefore exists if $t(\alpha) := h(\alpha) - \bar{e}_D(\alpha) = 0$ has an interior solution. The latter follows from continuity and the fact that $t(0) > 0 > t(1)$, as we now show. Assumption 3 implies the existence of $\alpha > 0$ such that $h(\alpha) > e_D^{nc}$. Because $h(\alpha)$ is strictly decreasing, the assumption therefore implies $h(0) > e_D^{nc} = \bar{e}_D(0)$. Thus, $t(0) = h(0) - \bar{e}_D(0) > 0$. Moreover, $t(1) = h(1) - \bar{e}_D(1) = h(1) < 0$.

(ii) $e_D^{ce} < e_D^{nc}$

Because $\bar{e}_D(0) = e_D^{nc}$, it suffices to show that $\bar{e}_D(\alpha)$ is strictly decreasing. Differentiating Equations (17) and (18) with respect to α,

$$\frac{de_D}{d\alpha} = -\frac{[C''(e_P)e_P\widehat{\beta}'(\alpha) + (1-\widehat{\beta}(\alpha))Q]\theta P(x_1,y_0)}{C''(e_P)C''(e_D) + [\theta P(x_1,y_0)(1-\widehat{\beta}(\alpha))]^2} < 0,$$

where $Q := (1-\theta)P(x_1) + \theta P(x_1,y_0)(1-e_D)\widehat{\beta}'(\alpha)$.

(iii) $e_P^{ce} < e_P^{nc}$

Comparing Equations (10) and (17), a sufficient condition for $e_P^{ce} < e_P^{nc}$ is $\widehat{\beta}(\alpha^{ce}) > e_D^{nc}$. We show that the latter holds. α^{ce} is the largest value of α such that

$$\widehat{\beta}(\alpha)(1-e_D^{nc}) \geq \frac{k_P(1-\theta)}{k_D\theta}\alpha.$$

Assumption 3 implies the existence of $\alpha_1 \in (0,1)$ such that

$$\widehat{\beta}(\alpha_1) - e_D^{nc} = \frac{k_P(1-\theta)}{k_D\theta}\alpha_1.$$

Hence,

$$\widehat{\beta}(\alpha_1)(1-e_D^{nc}) > \widehat{\beta}(\alpha_1) - e_D^{nc} = \frac{k_P(1-\theta)}{k_D\theta}\alpha_1.$$

Therefore, $\alpha^{ce} > \alpha_1$, implying $\widehat{\beta}(\alpha^{ce}) > \widehat{\beta}(\alpha_1)$ where $\widehat{\beta}(\alpha_1) > e_D^{nc}$. ■

Proof of Lemma 3. Maximizing $\widehat{\beta}(\alpha)k_D\theta(1-e_D^{re}) - \alpha k_P(1-\theta)$ is equivalent to choosing $\psi(z) \in \{0,1\}$ that maximizes

$$k_D\theta(1-e_D^{re})\int_Z \psi(z)f_M(z)dz - k_P(1-\theta)\int_Z \psi(z)f_S(z)dz.$$

The solution is $\psi(z) = 1$ if

$$k_D\theta(1-e_D^{re})f_M(z) \geq k_P(1-\theta)f_S(z) \tag{A4}$$

and $\psi(z)=0$ otherwise. In the problem (21), the constraint (22) is redundant as it is implied by Equation (23), which allows $\psi(z)=1$ only when Equation (A4) holds. Therefore, $\psi(z)=1$ whenever (A4) holds. ■

Proof of Proposition 3. We first prove the existence of an equilibrium with $e_D^{cr} \in (e_D^{ce}, e_D^{nc})$ as described. Next, we show that $e_P^{cr} < e_P^{nc}$. The proof that $e_P^{cr} > e_D^{cr}$ is similar to that in Proposition 1.

(i) Existence and $e_D^{cr} \in (e_D^{ce}, e_D^{nc})$

Let

$$\varphi(\alpha, e_D) := \widehat{\beta}(\alpha)k_D\theta(1-e_D) - \alpha k_P(1-\theta).$$

Then $\varphi(0, e_D)=0$ and $\varphi(1, e_D)<0$, where the inequality follows from Assumption 2. By Assumption 3, there exists $\alpha \in (0,1)$ such that $\varphi(\alpha, e_D^{nc})>0$. Therefore, because $\varphi(\alpha, e_D)$ is decreasing in e_D, there exists $\alpha \in (0,1)$ such that $\varphi(\alpha, e_D)>0$ when $e_D \leq e_D^{nc}$, implying that $\varphi(\alpha, e_D)$ is maximized by $\alpha \in (0,1)$ solving

$$\widehat{\beta}'(\alpha) = \frac{k_P(1-\theta)}{k_D\theta(1-e_D)}, e_D \leq e_D^{nc}. \tag{A5}$$

As in the proof of Proposition 2, let $\bar{e}_P(\alpha)$ and $\bar{e}_D(\alpha)$ denote the solution to Equations (17) and (18) given an arbitrary α; $\bar{e}_D(\alpha)$ is decreasing in α with $\bar{e}_D(0)=e_D^{nc}$. It follows that $e_D \leq e_D^{nc}$ at equilibrium and that (A5) must hold. Define

$$g(\alpha) \equiv 1 - \frac{k_P(1-\theta)}{\widehat{\beta}'(\alpha)k_D\theta} \tag{A6}$$

Using an argument similar to the one used for Proposition 2, an equilibrium is a solution to $t(\alpha) \equiv g(\alpha) - \bar{e}_D(\alpha) = 0$. With $h(\alpha)$ as defined in Equation (A3), the strict concavity of $\widehat{\beta}$ implies $g(\alpha) < h(\alpha)$ for $\alpha > 0$, while $g(0)=h(0)$. From (i) in the proof of Proposition 2, it follows that $t(0)>0$. Now,

$$t(\alpha^{ce}) = g(\alpha^{ce}) - \bar{e}_D(\alpha^{ce}) < h(\alpha^{ce}) - \bar{e}_D(\alpha^{ce}) = 0$$

Thus, there exists $\alpha^{cr} \in (0, \alpha^{ce})$ such that $t(\alpha^{cr})=0$. Because $\bar{e}_D(\alpha)$ is strictly decreasing, $e_D^{cr} = \bar{e}_D(\alpha^{cr}) > \bar{e}_D(\alpha^{ce}) = e_D^{ce}$.

(ii) $e_P^{cr} < e_P^{nc}$

We refer to the proof of Proposition 5 which shows that

$$\frac{\alpha^{cr}k_P(1-\theta)}{k_D\theta} + (1-e_D^{cr})(1-\widehat{\beta}(\alpha^{cr})) \leq 1 - e_D^{nc}. \tag{A7}$$

Here, we show that Equation (A7) together with $e_P^{cr} \geq e_P^{nc}$ is impossible. The latter inequality is equivalent to

$$(1-\theta)P(x_1)+\theta[P(y_1)+P(x_1,y_0)(1-e_D^{nc})]$$
$$\leq(1-\theta)P(x_1)(1-\alpha^{cr})+\theta[P(y_1)+P(x_1,y_0)(1-\widehat{\beta}(\alpha^{cr}))(1-e_D^{cr})]$$

or

$$-\frac{\alpha^{cr}P(x_1)(1-\theta)}{P(x_1,y_0)\theta}+(1-e_D^{cr})(1-\widehat{\beta}(\alpha^{cr}))\geq 1-e_D^{nc}. \tag{A8}$$

Both Equations (A7) and (A8) cannot simultaneously hold. ■

Proof of Lemma 4. It suffices to show that

$$\Pr(d=1,\omega_1)-\Pr(d=1,\omega_0)=e_P\Lambda(e_D,\alpha,\beta).$$

Now,

$$\Pr(d=1,\omega_i)/e_P=(1-\theta)P(x_1,\omega_i)(1-\alpha)+\theta P(y_1,\omega_i)$$
$$+\theta P(x_1,y_0,\omega_i)(1-e_D)(1-\beta).$$

Therefore,

$$[\Pr(d=1,\omega_1)-\Pr(d=1,\omega_0)]/e_P=(1-\theta)[P(x_1,\omega_1)-P(x_1,\omega_0)](1-\alpha)$$
$$+\theta P(y_1,\omega_1)-\theta P(x_1,y_0,\omega_0)(1-e_D)(1-\beta),$$

which equals $\Lambda(e_D,\alpha,\beta)$. ■

Proof of Proposition 4. The proof follows from the argument in the text and from $e_P^{ce}<e_P^{nc}$, shown in Proposition 2.

Proof of Proposition 5. We show that $\Lambda^{cr}>\Lambda^{nc}$; note that this implies (Equation (A7)) in the proof of Proposition 3. Let $\alpha(e_D)$ maximize $\Lambda(e_D,\alpha,\widehat{\beta}(\alpha))$ with respect to α. By the envelope theorem,

$$\frac{d\Lambda[e_D,\alpha(e_D),\widehat{\beta}(\alpha(e_D))]}{de_D}=k_D\theta>0.$$

Therefore, and recalling that α^{cr} maximizes $\Lambda(e_D^{cr},\alpha,\widehat{\beta}(\alpha))$,

$$\Lambda^{cr}=\Lambda[e_D^{cr},\alpha(e_D^{cr}),\widehat{\beta}(\alpha(e_D^{cr}))]>\Lambda[0,\alpha(0),\widehat{\beta}(\alpha(0))].$$

Next, we show that $\Lambda(0,\alpha^*,\widehat{\beta}(\alpha^*))>\Lambda(e_D^{nc},0,0)$ where α^* is short-hand for $\alpha(0)$. From the first-order condition,

$$\widehat{\beta}'(\alpha^*)=\frac{k_P(1-\theta)}{k_D\theta}.$$

Assumption 3 implies the existence of an interval $(\alpha_0,\alpha_1)\subset(0,1)$ such that

$$\widehat{\beta}(\alpha) = e_D^{nc} + \frac{k_P(1-\theta)}{k_D\theta}\alpha \text{ if } \alpha = \alpha_0 \text{ or } \alpha = \alpha_1.$$

Therefore, $\alpha^* \in (\alpha_0, \alpha_1)$ and

$$\widehat{\beta}(\alpha^*) > e_D^{nc} + \frac{k_P(1-\theta)}{k_D\theta}\alpha^*. \tag{A9}$$

Consider now e_D^* solving $\Lambda(0, \alpha^*, \widehat{\beta}(\alpha^*)) = \Lambda(e_D^*, 0, 0)$. Straightforward substitutions yield

$$\widehat{\beta}(\alpha^*) = e_D^* + \frac{k_P(1-\theta)}{k_D\theta}\alpha^*.$$

Hence, $e_D^* > e_D^{nc}$ and therefore $\Lambda(e_D^*, 0, 0) > \Lambda(e_D^{nc}, 0, 0)$, which proves the claim. ■

APPENDIX B

We discuss two variants of the baseline setting. First, we consider the case where the defendant knows the true fact. Next, we discard the possibility that a cross-examiner knowing that the plaintiff did not conceal evidence nevertheless attempts to persuade the arbitrator of the contrary.

Informed defendant

The defendant's investigation effort will now depend on the true fact, which we write as $e_D(\omega)$. Consider the procedure without the opportunity of cross-examination. When $\omega = \omega_1$, D knows that uncovering y_0 to counter x_1 is impossible. Therefore, $e_D(\omega_1) = 0$. When $\omega = \omega_0$, the situation where P reports x_1 and y_0 will be uncovered if the state of the evidence is M arises with probability $e_P\theta P(x_1 \mid \omega_0)$. Therefore, $e_D(\omega_0)$ solves $C'(e_D(\omega_0)) = e_P\theta(1-q)$. The plaintiff's investigation effort solves (10) but with $e_D(\omega_0)$ in lieu of e_D. The foregoing yields the equilibrium investigation efforts under no cross-examination, e_P^{nc}, $e_D^{nc}(\omega_0)$ and $e_D^{nc}(\omega_1)$.

We reformulate Assumption 3 as

$$\alpha k_P(1-\theta) < k_D\theta(\beta - e_D^{nc}(\omega_0)) \text{ for some } \beta \leq \widehat{\beta}(\alpha), \alpha \in (0,1).$$

All of the previous results then hold with $e_D(\omega_0)$ in lieu of e_D in the equations. Thus, e_P and $e_D(\omega_0)$ solve

$$C'(e_P) = (1-\theta)P(x_1)(1-\alpha) + \theta[P(y_1) + P(x_1, y_0)(1 - e_D(\omega_0))(1-\beta)],$$

$$C'(e_D(\omega_0)) = e_P\theta(1-\beta)(1-q),$$

α solves

$$\frac{\widehat{\beta}(\alpha)}{\alpha} = \frac{k_P(1-\theta)}{k_D\theta(1 - e_D(\omega_0))}$$

under cross-examination only and solves

$$\widehat{\beta}'(\alpha) = \frac{k_P(1-\theta)}{k_D\theta(1-e_D(\omega_0))}$$

when re-examination is allowed. Similar to Lemma 4, the probability of correct adjudication is $\bar{u}_A = 1 - p + e_P\Lambda(e_D(\omega_0), \alpha, \beta)$ where

$$\Lambda(e_D(\omega_0), \alpha, \beta) := p\theta + [k_P(1-\theta)(1-\alpha) - k_D\theta(1-e_D(\omega_0))(1-\beta)],$$

and where $e_D(\omega_0)$, α and $\beta = \widehat{\beta}(\alpha)$ depend on the procedure. The Propositions 4 and 5 hold unchanged.

Honest cross-examiner

Suppose that the defendant and his counsel share the same information. If the defendant uncovered the evidence, the pair of reports $r_P = (x_1, \emptyset)$ and $r_D = \emptyset$ arises only if the state of the evidence is S. We now assume that the defendant's counsel would then not cross-examine the plaintiff for the purpose of persuading the arbitrator that the state of the evidence is M, as this would be unethical. Thus, the plaintiff is cross-examined only when the defendant does not uncover the evidence, which occurs with probability $1 - e_D$.

When $r_P = (x_1, \emptyset)$ and $r_D = \emptyset$, the observation that there is no cross-examination reveals to the arbitrator that the state of the evidence is S. The arbitrator's belief is then $\mu((x_1, \emptyset), \emptyset, \chi = \emptyset) = P(\omega_1 \mid x_1) > \frac{1}{2}$, where $\chi = \emptyset$ denotes that the plaintiff is not cross-examined. When the plaintiff is cross-examined, the arbitrator knows that the defendant did not uncover the evidence. Lemma 2 is now replaced with $\mu((x_1, \emptyset), \emptyset, \chi = b) \le \frac{1}{2}$ if and only if $\alpha k_P(1-\theta) \le \beta k_D\theta$. Under cross-examination only, α^{ce} then solves

$$\widehat{\beta}'(\alpha^{ce}) = \frac{k_P(1-\theta)}{k_D\theta}. \tag{B1}$$

Under cross and re-examination, α^{cr} solves

$$\widehat{\beta}'(\alpha^{cr}) = \frac{k_P(1-\theta)}{k_D\theta}. \tag{B2}$$

Comparing with Equation (16), the probability that the plaintiff prevails conditional on uncovering the evidence is now

$$\begin{aligned} W(e_D, \alpha, \beta) &= (1-\theta)P(x_1)[e_D + (1-e_D)(1-\alpha)] \\ &\quad + \theta[P(y_1) + P(x_1, y_0)(1-e_D)(1-\beta)]. \end{aligned}$$

As before, e_P solves $C'(e_P) = W(e_D, \alpha, \beta)$, but now e_D solves

$$C'(e_D) \ge \theta P(x_1, y_0)(1-\beta) - (1-\theta)P(x_1)\alpha,$$

where $e_D > 0$ only if the right-hand side is positive (for some values of the parameters the defendant will not investigate). Except for the new equilibrium values of α, e_P, and e_D, the Propositions 2 and 3 hold unchanged.

The value of communication now becomes

$$\Lambda(e_D, \alpha, \beta) = p\theta + [k_P(1-\theta)(1-\alpha(1-e_D)) - k_D\theta(1-e_D)(1-\beta)].$$

Using Equations (B1) and (B2), it is then easily shown that the Propositions 4 and 5 hold unchanged.

REFERENCES

Blackwell, D., and M.A. Girschick. 1954. *Theory of Games and Statistical Decisions*. New York: Wiley & Sons.

Bergemann, D., and S. Morris. 2019. “Information Design: A Unified Perspective,” 57 *Journal of Economic Literature* 1–57.

Bull, J., and J. Watson. 2019. “Statistical Evidence and the Problem of Robust Litigation,” 50 *RAND Journal of Economics* 974–1003.

Daughety, A., and J. Reinganum. 2000. “On the Economics of Trials: Adversarial Process, Evidence, and Equilibrium Bias,” 16 *Journal of Law, Economics, and Organization* 365–94.

Demougin, D., and C. Fluet. 2008. “Rules of Proof, Courts, and Incentives,” 39 *RAND Journal of Economics* 20–40.

Dewatripont, M., and J. Tirole. 1999. “Advocates,” 107 *Journal of Political Economy* 1–39.

Dowling, S.C. 2021. “(Un)Due Process: Adversarial Cross-Examination in Title IX Adjudications,” 73 *Maine Law Review* 123–67.

Dye, R.A. 1985. “Disclosure of Nonproprietary Information,” 23 *Journal of Accounting Research* 123–45.

Emons, W., and C. Fluet. 2009. “Accuracy versus Falsification Costs: The Optimal Amount of Evidence Under Different Procedures,” 25 *Journal of Law, Economics, and Organization* 134–56.

Farrell, J. 1986. “Voluntary Disclosure: Robustness of the Unraveling Result, and Comments on Its Importance,” in R.E. Grieson, ed., *Antitrust and Regulation*, 91–103. Lexington, MA: Lexington.

Fluet, C., and M. Mungan. 2025. “Oriented Data Generating Processes: A Categorization of ROC Curves,” *Theory & Decision*. https://doi.org/10.1007/s11238-025-10083-7. Advance publication.

Frankel, M.E. 1975. “The Search for Truth: An Umpireal View,” 123 *University of Pennsylvania Law Review* 1031–9.

Frankel, M.E. 1980. *Partisan Justice*. New York, NY: Hill & Wang.

Gentzkow, M., and E. Kamenica. 2017a. “Bayesian Persuasion with Multiple Senders and Rich Signal Spaces,” 104 *Games and Economic Behavior* 411–29.

Gentzkow, M., and E. Kamenica. 2017b. “Competition in Persuasion,” 84 *Review of Economic Studies* 300–22.

Green, J., and N. Stokey. 1978. “Two Representations of Information Structures and their Comparisons,” Technical report no. 271. Institute for Mathematical Studies in the Social Sciences, Stanford University, Stanford.

Grossman, S.J. 1981. “The Informational Role of Warranties and Private Disclosure about Product Quality,” 24 *Journal of Law and Economics* 461–83.

Hosmer, D.W., and S. Lemeshow. 2000. *Applied Logistic Regression*. 2nd ed. New York: John Wiley & Sons.

Kamenica, E. 2019. “Bayesian Persuasion and Information Design,” 11 *Annual Review of Economics* 249–72.

Kamenica, E., and M. Gentzkow. 2011. “Bayesian Persuasion,” 101 *American Economic Review* 2590–615.

Kaplow, L., and S. Shavell. 1989. “Legal Advice about Information to Present in Litigation: Its Effects and Social Desirability,” 102 *Harvard Law Review* 567–615.

Kartik, N., F.L. Lee, and W. Suen. 2017. “Investment in Concealable Information by Biased Experts,” 48 *RAND Journal of Economics* 24–43.

Katz, A. 1988. “Judicial Decision Making and Litigation Expenditure,” 8 *International Review of Law and Economics* 127–43.

Kim, C. 2014. “Adversarial and Inquisitorial Procedure in Information Acquisition,” 30 *Journal of Law, Economics, and Organization* 767–803.

Langbein, J.H. 1985. “The German Advantage in Civil Procedure,” 52 *University of Chicago Law Review* 823–66.

Lehmann, E.L, and J.P. Romano. 2005. *Testing Statistical Hypotheses*. 3rd ed. New York: Springer.

Li, F., and P. Norman. 2021. “Sequential Persuasion,” 16 *Theoretical Economics* 639–75.

Matthews, S., and A. Postlewaite. 1985. “Quality Testing and Disclosure,” 16 *RAND Journal of Economics* 328–40.
Milgrom, P. 1981. “Good News and Bad News: Representation Theorems and Applications,” 12 *Bell Journal of Economics* 380–91.
Milgrom, P., and J. Roberts. 1986. “Relying on the Information of Interested Parties,” 17 *RAND Journal of Economics* 18–32.
Perez-Richet, E. 2014. “Interim Bayesian Persuasion: First Steps,” 104 *American Economic Review: Papers & Proceedings* 469–74.
Posner, R.A. 1999. “An Economic Approach to the Law of Evidence,” 51 *Stanford Law Review* 1477–546.
Schweizer, U. 2017. “Incentives to Acquire Information under Mandatory versus Voluntary Disclosure,” 33 *Journal of Law, Economics, and Organization* 173–92.
Shavell, S. 1989. “A Note on the Incentive to Reveal Information,” 14 *Geneva Papers on Risk and Insurance* 66–74.
Shavell, S. 1994. “Acquisition and Disclosure of Information Prior to Sale,” 25 *RAND Journal of Economics* 20–36.
Shin, H. 1998. “Adversarial and Inquisitorial Procedures in Arbitration,” 29 *RAND Journal of Economics* 378–405.
Sobel, J. 1985. “Disclosure of Evidence and Resolution of Disputes: Who Should Bear the Burden of Proof,” in A. Roth, ed., *Game-Theoretic Models of Bargaining*, 351–59. Cambridge, UK: Cambridge University Press.
Weigend, T. 2010. “Should We Search for the Truth, and Who Should Do It,” 36 *North Carolina Journal of International Law and Commercial Regulation* 389–416.
Wickelgren, A. 2010. “A Right to Silence for Civil Defendants?” 26 *Journal of Law, Economics and Organization* 92–114.
Wigmore, J.H. 1940. *A Treatise on the Anglo-American System of Evidence*. 3rd ed. Boston, MA: Little Brown and Co.

The Journal of Law, Economics, and Organization, 2026, 00, 1–31
https://doi.org/10.1093/jleo/ewaf020
Article